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On the influence of heat transfer in peristalsis with variable viscosity

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ABSTRACT

This study concentrates on the heat transfer characteristics and endoscope effects for peristaltic flow of a third order fluid. Two models of variable viscosity are chosen. Both perturbation and numerical solutions are obtained in each case. A comparative study is also made between the two solutions. The importance of pertinent flow parameters entering into the flow modeling is discussed.

1. Introduction

Peristaltic flows are relevant to a number of industrial and physiological applications including urine transport from kidney to the bladder, swallowing of food through the esophagus, chyme movement in the gastrointestinal tract, vasomotion of small blood vessels such as arteries, venules and capillaries, in roller and finger pumps and many others. Such flows are extensively studied in various geometries by using different assumptions of large wavelength, small amplitude ratio, small wave number, creeping flow etc. At present a wealth of literature on the topic dealing with the peristalsis in viscous and non-Newtonian fluids is available. However, few recent studies [1–16] and many refs there in may be mentioned in this direction.

Existing literature indicates that little efforts are made to explain the heat transfer effects on the peristalsis. Radhakrishnamacharya and Srincvasulu [17] discussed the combined effects of wall properties and heat transfer on the peristaltic flow of a viscous fluid in a channel. Mekheimer and Elmaboud [18] analyzed the heat transfer and MHD effects on the peristaltic transport of a viscous fluid in a vertical annulus. Srinivas and Kothandapani [19] studied the peristaltic flow with heat transfer in an asymmetric channel when the wavelength is very large. In another attempts, Kothandapani and Srinivas [20] analyzed the influence of elasticity of the flexible walls on MHD peristaltic flow of a viscous fluid in a porous channel with heat transfer. Very recently, Hayat et al. [21] examined the MHD peristaltic flow of a third order fluid with an endoscope and variable viscosity.

The purpose of the present investigation is to perform a study which can describe the heat transfer and endoscope effects on the peristaltic flow of a third order fluid when the viscosity is not constant. Two cases of variable viscosity are taken into account. The flow analysis is performed under the assumtion of long wavelength situation. Series solution is derived for small Deborah number. Numerical solution is obtained by employing shooting method. Comparison between the two presented solution is given and discussed.

2. Development of the problem

We investigate the flow of a third order fluid through the gap between two concentric tubes. The inner tube is maintained at a temperature T_0. A sinusoidal wave travels down on the wall of the outer tube having temperature T_1. We choose cylindrical coordinate system in such a way that $\overline{R}$ is in the radial direction and the $\overline{Z}$ along the centre line of the inner and outer tubes. Shapes of the two walls are described by the following expressions

$$\overline{R}_1 = a_1, \tag{1}$$

$$\overline{R}_2 = a_2 + b\sin\frac{2\pi}{\lambda}(\overline{Z} - c\bar{t}). \tag{2}$$

In above expressions a_1 is the radius of the inner tube, a_2 is the radius of the outer tube at the inlet, b is the wave amplitude, λ is the wavelength, c is the wave speed and $\bar{t}$ is the time.

In the fixed frame $(\overline{R}, \overline{Z})$ the continuity, momentum and energy equations give

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$$\frac{\partial \overline{U}}{\partial \overline{R}}+\frac{\overline{U}}{\overline{R}}+\frac{\partial \overline{W}}{\partial \overline{Z}}=0, \tag{3}$$

$$\rho\left(\frac{\partial}{\partial \bar{t}}+\overline{U}\frac{\partial}{\partial \overline{R}}+\overline{W}\frac{\partial}{\partial \overline{Z}}\right)\overline{U}=-\frac{\partial \bar{p}}{\partial \overline{R}}+\frac{1}{\overline{R}}\frac{\partial}{\partial \overline{R}}\left(\overline{R}\overline{S}_{\overline{RR}}\right)+\frac{\partial}{\partial \overline{Z}}(\overline{S}_{\overline{RZ}})-\frac{\overline{S}_{\bar{\theta}\bar{\theta}}}{\overline{R}}, \tag{4}$$

$$\rho\left(\frac{\partial}{\partial \bar{t}}+\overline{U}\frac{\partial}{\partial \overline{R}}+\overline{W}\frac{\partial}{\partial \overline{Z}}\right)\overline{W}=-\frac{\partial \bar{p}}{\partial \overline{Z}}+\frac{1}{\overline{R}}\frac{\partial}{\partial \overline{R}}\left(\overline{R}\overline{S}_{\overline{RZ}}\right)+\frac{\partial}{\partial \overline{Z}}\left(\overline{S}_{\overline{ZZ}}\right), \tag{5}$$

$$\rho c_p\left(\frac{\partial}{\partial \bar{t}}+\overline{U}\frac{\partial}{\partial \overline{R}}+\overline{W}\frac{\partial}{\partial \overline{Z}}\right)\overline{T}=\overline{S}_{\overline{RR}}\frac{\partial \overline{U}}{\partial \overline{R}}+\overline{S}_{\overline{RZ}}\frac{\partial \overline{W}}{\partial \overline{R}}+\overline{S}_{\overline{ZR}}\frac{\partial \overline{U}}{\partial \overline{Z}}+\overline{S}_{\overline{ZZ}}\times\frac{\partial \overline{W}}{\partial \overline{Z}}+k\left(\frac{\partial^2 \overline{T}}{\partial \overline{R}^2}+\frac{1}{\overline{R}}\frac{\partial \overline{T}}{\partial \overline{R}}+\frac{\partial^2 \overline{T}}{\partial \overline{Z}^2}\right), \tag{6}$$

where $\bar{p}$ is the pressure and $\overline{U}$, $\overline{W}$ are the respective velocity components in the radial and axial directions respectively, $\overline{T}$ is the temperature, ρ is the density, k denotes the thermal conductivity and c_p is the specific heat at constant pressure.

Expressions of an extra stress tensor $\overline{S}$ in a third order fluid is given by the following relation

$$\overline{S}=\left(\mu+\beta_3 tr\overline{A}_1^2\right)\overline{A}_1+\alpha_1\overline{A}_2+\alpha_2\overline{A}_1^2+\beta_1\overline{A}_3+\beta_2\left(\overline{A}_1\overline{A}_2+\overline{A}_2\overline{A}_1\right). \tag{7}$$

where μ, $\alpha_i(i=1,2)$ and $\beta_i(i=1,2,3)$ are the material constants satisfying the thermodynamical conditions defined in [13]. The first three Rivlin–Ericksen tensors can be written as

$$\overline{A}_1=\overline{L}+\overline{L}^t,$$

$$\overline{A}_{n+1}=\frac{d\overline{A}_n}{dt}+\overline{A}_n\overline{L}+\overline{L}^t\overline{A}_n,\quad n=1,2$$

in which $\overline{L}=grad\overline{V}$ and t is the matrix transpose.

The coordinates in the fixed $(\overline{R},\overline{Z})$ and wave $(\bar{r},\bar{z})$ frames are related through the following transformations

$$\bar{r}=\overline{R},\bar{z}=\overline{Z}-c\bar{t},\qquad \bar{u}=\overline{U},\overline{w}=\overline{W}-c, \tag{8}$$

where $\bar{u}$ and $\overline{w}$ are the velocities in the wave frame.

The boundary conditions in the wave frame are

$$\overline{w}=-c,\quad \bar{u}=0 \text{ at } \bar{r}=\bar{r}_1=\varepsilon, \tag{9a}$$

$$\overline{w}=-c,\quad \text{at } \bar{r}=\bar{r}_2=a_2+b\sin\frac{2\pi}{\lambda}\bar{z}, \tag{9b}$$

$$\overline{T}=\overline{T}_0,\quad \text{at } \bar{r}=\bar{r}_1, \tag{9c}$$

$$\overline{T}=\overline{T}_1,\quad \text{at } \bar{r}=\bar{r}_2. \tag{9d}$$

If $\delta, P_r, Re, E_c, \varepsilon, \left(\phi=\frac{b}{a_2}<1\right), \mu_0$, are respectively, the wave number, Prandtl number, Reynold number, Eckert number, radius ratio and reference viscosity then defining

$$R=\frac{\overline{R}}{a_2},\quad r=\frac{\bar{r}}{a_2},\quad Z=\frac{\overline{Z}}{\lambda},\quad z=\frac{\bar{z}}{\lambda},\quad W=\frac{\overline{W}}{c},\quad w=\frac{\overline{w}}{c},$$
$$U=\frac{\lambda\overline{U}}{a_2c},\quad u=\frac{\lambda\bar{u}}{a_2c},\quad P=\frac{a_2^2\overline{P}}{c\lambda\mu_0},\quad \theta=\frac{(\overline{T}-\overline{T}_1)}{(\overline{T}_0-\overline{T}_1)},\Pr=\frac{\mu_0c_p}{k},$$
$$\mu(r)=\frac{\bar{\mu}(\bar{r})}{\mu_0},\quad t=\frac{c\bar{t}}{\lambda},\quad \delta=\frac{a_2}{\lambda},\ Re=\frac{\rho c a_2}{\mu_0},\ S=\frac{a_2\overline{S}}{c\mu_0},\ \phi=\frac{b}{a_2},$$
$$r_1=\frac{\bar{r}_1}{a_1}=\varepsilon,\ r_2=\frac{\bar{r}_2}{a_2}=1+\phi\sin(2\pi z),\ E_c=\frac{c^2}{c_p(T_0-T_1)}. \tag{10}$$

Eqs. (3)–(7) and (9) become

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z}=0, \tag{11}$$

$$Re\delta^3\left(u\frac{\partial}{\partial r}+w\frac{\partial}{\partial z}\right)u=-\frac{\partial p}{\partial r}+\frac{\delta}{r}\frac{\partial}{\partial r}(rS_{rr})+\delta^2\frac{\partial}{\partial z}(S_{rz})-\frac{\delta S_{\theta\theta}}{r}, \tag{12}$$

$$Re\delta\left(u\frac{\partial}{\partial r}+w\frac{\partial}{\partial z}\right)w=-\frac{\partial p}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}(rS_{rz})+\delta\frac{\partial}{\partial z}(S_{zz}), \tag{13}$$

$$Re\delta\Pr\left(u\frac{\partial}{\partial r}+w\frac{\partial}{\partial z}\right)\theta=E_c\Pr\left(\delta\frac{\partial u}{\partial r}S_{rr}+\frac{\partial w}{\partial r}S_{rz}+\delta^2\frac{\partial u}{\partial z}S_{zr}+S_{zz}\frac{\partial w}{\partial z}\delta\right)+\frac{\partial^2\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\theta}{\partial r}+\delta^2\frac{\partial^2\theta}{\partial z^2}, \tag{14}$$

$$w=-1,\ \text{at } r=r_1=\varepsilon, \tag{15a}$$
$$w=-1,\ \text{at } r=r_2=1+\phi\sin(2\pi z), \tag{15b}$$
$$\theta=1,\ \text{at } r=r_1, \tag{15c}$$
$$\theta=0,\ \text{at } r=r_2 \tag{15d}$$

and

$$S_{rr}=(2\lambda_1+\lambda_2)\left(\frac{\partial w}{\partial r}\right)^2, \tag{16a}$$

$$S_{rz}=\left[\mu(r)\left(\frac{\partial w}{\partial r}\right)+2\Gamma\left(\frac{\partial w}{\partial r}\right)^3\right], \tag{16b}$$

$$S_{zz}=\lambda_2\left[\left(\frac{\partial w}{\partial r}\right)^2\right], \tag{16c}$$

$$S_{\theta\theta}=\left(\mu(r)-\frac{\partial u}{\partial r}\lambda_1\right)\frac{u}{r}+\left(5\lambda_1+4\lambda_2+4\Gamma\left(\frac{\partial w}{\partial r}\right)^2\right)\left(\frac{u}{r}\right)^2+8\Gamma\left(\frac{\partial u}{\partial r}\right)^2, \tag{16d}$$

where $\lambda_1=\alpha_1c/\mu_0a_2$, $\lambda_2=\alpha_2c/\mu_0a_2$ and the Deborah number $\Gamma=\beta c^2/\mu_0a_2$.

Invoking Eqs. (16a)–(16c) and using long wavelength and low Reynolds number assumptions, Eqs. (11)–(13) yield

$$0=\frac{\partial p}{\partial r}, \tag{17}$$

$$0=-\frac{\partial p}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\left(\mu(r)\left(\frac{\partial w}{\partial r}\right)+2\Gamma\left(\frac{\partial w}{\partial r}\right)^3\right)\right), \tag{18}$$

$$0=B_r\left(\left(\frac{\partial w}{\partial r}\right)\left(\mu(r)\left(\frac{\partial w}{\partial r}\right)+2\Gamma\left(\frac{\partial w}{\partial r}\right)^3\right)\right)+\frac{\partial^2\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\theta}{\partial r}, \tag{19}$$

where $B_r=E_c$ Pr is the Brinkman number and Eq. (17) shows that $p\neq p(r)$.

3. Solution of the problem

3.1. Case I

In order to seek the solution of the problem we let

$$\mu(r)=e^{-\alpha r} \tag{20}$$

which by Maclaurin series can be written as

$$\mu(r)=1-\alpha r+O(\alpha^2). \tag{21}$$

Substituting above equation into Eqs. (15), (18) and (19) and then solving the resulting problem by perturbation technique one obtains

$$w = -1 + \frac{dp}{dz}\frac{1}{12}(3r^2 + 2\alpha r^3 + 12L_3(\ln r + \alpha r) + 12L_4) + \Gamma\bigg(L_{56}r^8 + L_{57}r^7 + L_{58}r^6 + L_{59}r^5 + L_{60}r^4 + L_{61}r^3 + L_{62}r^2 + L_{64}r + L_{55} + L_{65}\ln r - \frac{L_{49}}{r} + \frac{L_{63}}{r^2}\bigg), \tag{22}$$

$$\theta = L_{18}r^7 + L_{19}r^6 + L_{20}r^5 + L_{21}r^4 + L_{22}r^3 + L_{23}r^2 + L_{16}r + L_{24}(\ln r)^2 + L_{27}\ln r + L_{28} + \Gamma\Big(L_{120}r^{17} + L_{121}r^{16} + L_{122}r^{15} + L_{123}r^{14} + L_{124}r^{13} + L_{125}r^{12} + L_{126}r^{11} + L_{127}r^{10} + L_{128}r^9 + L_{129}r^8 + L_{130}r^7 + L_{131}r^6 + L_{132}r^5 + L_{133}r^4 + L_{134}r^3 + L_{135}r^2 + L_{116}r + L_{136}(\ln r)^2 + \frac{L_{118}}{r} + \frac{L_{137}}{r^2} + L_{140}\ln r + L_{141}, \tag{23}$$

$$\frac{dp}{dz} = \frac{Q + (r_2^2 - r_1^2)}{L_{66}} + \Gamma\left(\frac{-L_{67}}{L_{66}}\right), \tag{24}$$

where Γ is used as the perturbation quantity. All the appearing quantities in above equations are included in the Appendix A.

The dimensionless pressure rise Δp_λ and friction force on outer tube $F_\lambda^{(o)}$ and inner tube $F_\lambda^{(i)}$, are defined by

$$\Delta p_\lambda = \int_0^1 \frac{dp}{dz}dz, \tag{25}$$

$$F_\lambda^{(o)} = \int_0^1 r_1^2\left(-\frac{dp}{dz}\right)dz, \tag{26}$$

$$F_\lambda^{(i)} = \int_0^1 r_2^2\left(-\frac{dp}{dz}\right)dz, \tag{27}$$

in which dp/dz is defined through Eq. (24).

3.2. Case II

Here we are interested in the temperature-dependent viscosity. For that we choose Reynold's model of viscosity i.e.,

$$\mu(\theta) = e^{-\beta\theta}, \tag{28}$$

where β is a parameter.

Employing the same methodology as in Case I, the numerical solution of axial velocity is presented in Subsection 3.2. However the series solutions are

$$w = -1 + \frac{dp}{dz}\left(\frac{r^2}{4} + \beta a_4(r) - \frac{a_{11}}{a_{12}}(\ln r + \beta a_5(r) - a_{13})\right) + \Gamma\left(-\frac{a_{14}}{a_{11}}\ln r + a_7(r) - \beta a_5(r) + a_{15}\right), \tag{29}$$

$$\frac{dp}{dz} = \frac{2Q + (r_2^2 - r_1^2)}{2a_8(r)}, \tag{30}$$

$$\theta = -B_r(a_{14}(r) + a_{15}(r) + a_{20}\ln r + a_{21}) + \Gamma(-B_r(a_{24}(r) + a_{25}(r))). \tag{31}$$

where all $a_i(r)$ and a_{ij} $(i,j = 1,2,3\ldots)$ are given in Appendix A. (see Fig. 1)

4. Comparison between numerical and perturbation solutions

Here the problem consisting of Eqs. (15a,b) and (18) is also solved numerically by employing shooting method. The numerical

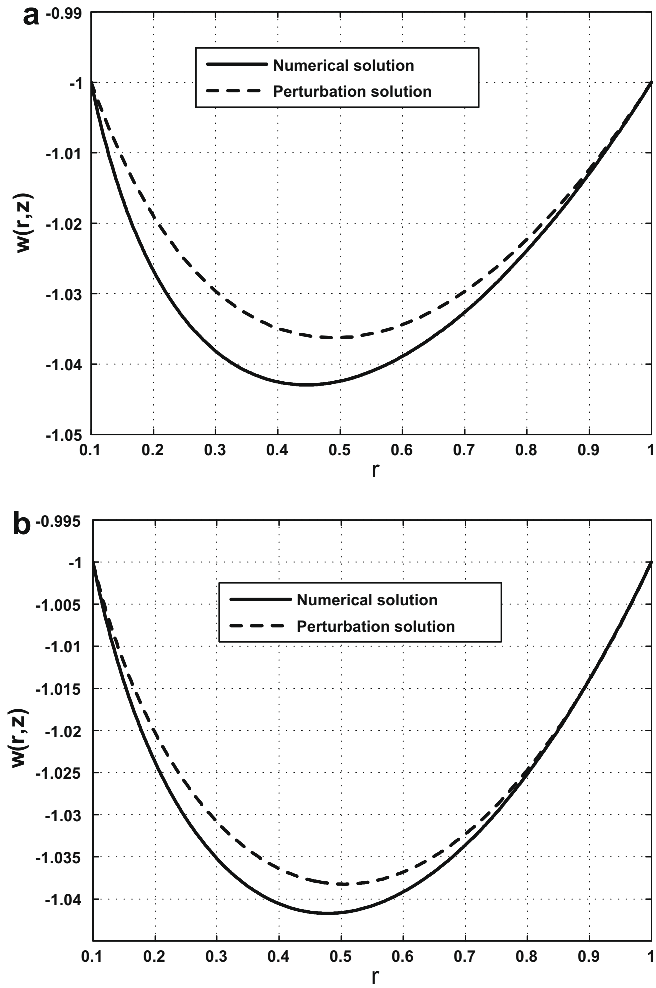

Fig. 1. (a) Comparison of axial velocity for perturbation and numerical solutions when $\Gamma = 0.005$, $\alpha = 0.2$, $\phi = 0.6$, $\varepsilon = 0.1$, $\frac{dp}{dz} = 0.4$, $z = 0.5$. (b) Comparison of axial velocity for perturbation and numerical solutions when $\Gamma = 0.005$, $\beta = 0.2$, $\phi = 0.6$, $\varepsilon = 0.1$, $\frac{dp}{dz} = 0.4$, $z = 0.5$, $B_r = 0.1$.

Table 1
Case I.

r	Numerical solution	Perturbation solution	Error
0.10	−1.000000	−1.000000	0.000000
0.15	−1.016465	−1.010710	0.005755
0.20	−1.029170	−1.018960	0.010020
0.25	−1.037464	−1.025111	0.012050
0.30	−1.042150	−1.031621	0.010206
0.35	−1.045808	−1.035891	0.009573
0.40	−1.047766	−1.038451	0.008970
0.45	−1.048315	−1.039621	0.008362
0.50	−1.048003	−1.037822	0.009809
0.55	−1.046604	−1.038521	0.007783
0.60	−1.044183	−1.037642	0.006303
0.65	−1.041439	−1.029661	0.011438
0.70	−1.037320	−1.028631	0.008447
0.75	−1.033230	−1.025292	0.007421
0.80	−1.027559	−1.022292	0.005152
0.85	−1.022216	−1.018654	0.004967
0.90	−1.015081	−1.014380	0.000691
0.95	−1.007172	−1.006592	0.000576
1.00	−1.000000	−1.000000	0.000000

Table 2
Case II.

r	Numerical solution	Perturbation solution	Error
0.10	−1.000000	−1.000000	0.000000
0.15	−1.013079	−1.014210	0.001115
0.20	−1.020100	−1.021160	0.000156
0.25	−1.026625	−1.030789	0.004039
0.30	−1.030679	−1.031621	0.000913
0.35	−1.045808	−1.038601	0.006939
0.40	−1.036289	−1.040463	0.008970
0.45	−1.037758	−1.041561	0.003652
0.50	−1.038221	−1.041632	0.002408
0.55	−1.037908	−1.040785	0.003952
0.60	−1.036671	−1.037642	0.006303
0.65	−1.034967	−1.036805	0.000935
0.70	−1.032094	−1.033389	0.001253
0.75	−1.029015	−1.029900	0.000859
0.80	−1.024494	−1.024958	0.000452
0.85	−1.020031	−1.020021	0.000001
0.90	−1.013829	−1.014380	0.000543
0.95	−1.007942	−1.007858	0.000083
1.00	−1.000000	−1.000000	0.000000

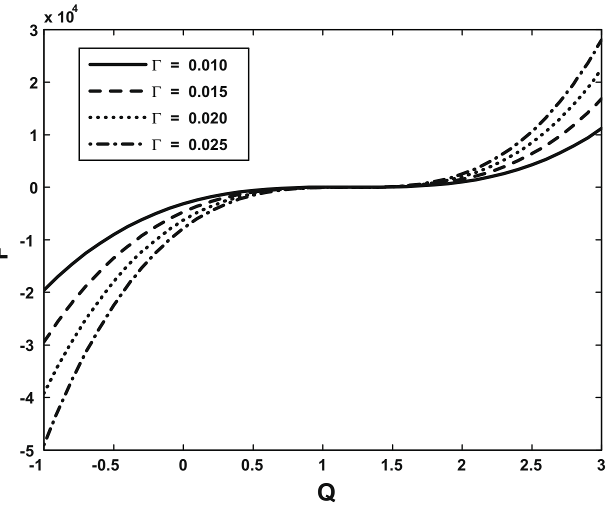

Fig. 4. Frictional force (on inner tube) versus flow rate when $\varepsilon = 0.3$, $\phi = 0.9$, $\alpha = 0.8$.

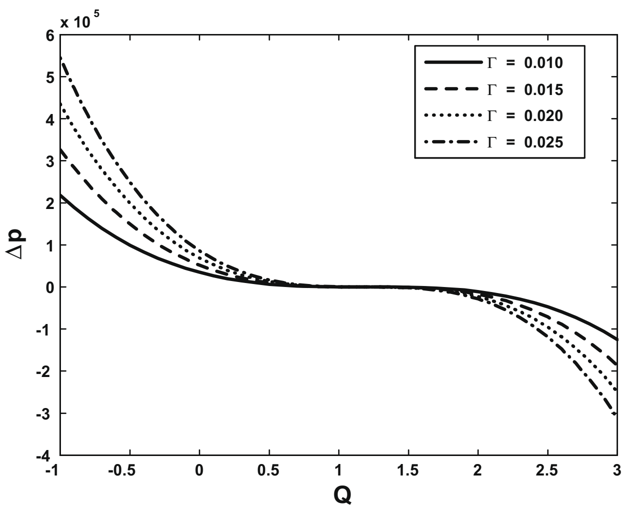

Fig. 2. Pressure rise versus flow rate when $\varepsilon = 0.3$, $\phi = 0.9$, $\alpha = 0.8$.

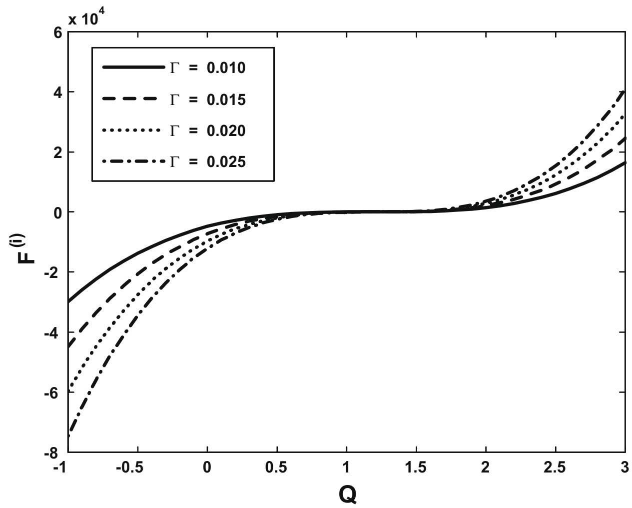

Fig. 5. Frictional force (on outer tube) versus flow rate when $\varepsilon = 0.3$, $\phi = 0.9$, $\alpha = 0.8$.

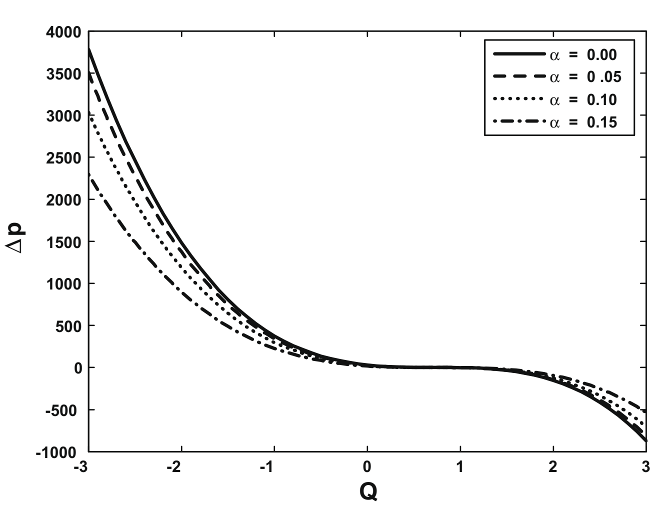

Fig. 3. Pressure rise versus flow rate when $\varepsilon = 0.8$, $\phi = 0.85$, $\Gamma = 0.8$.

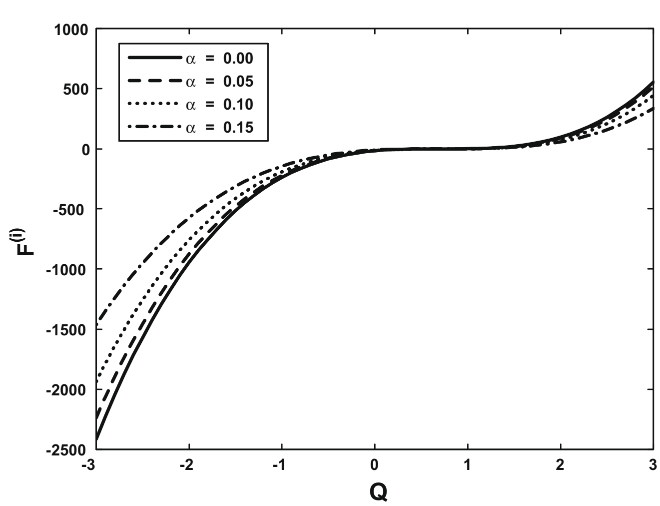

Fig. 6. Frictional force (on outer tube) versus flow rate when $\varepsilon = 0.8$, $\phi = 0.85$, $\Gamma = 0.003$.

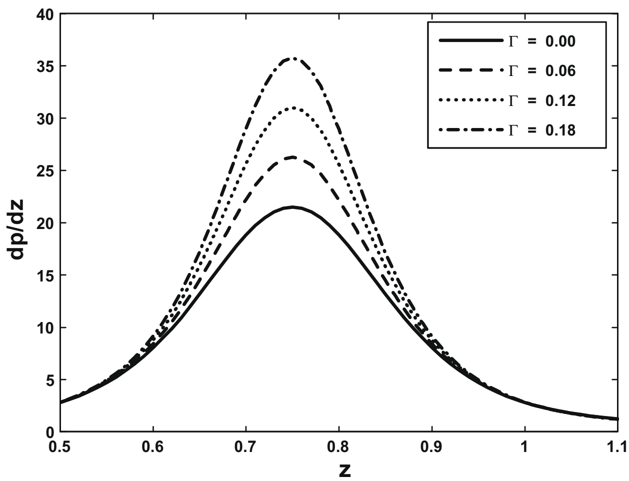

Fig. 7. Axial pressure gradient dp/dz when $\varepsilon = 0.3$, $\phi = 0.4$, $\alpha = 0.2$, $Q = 3$.

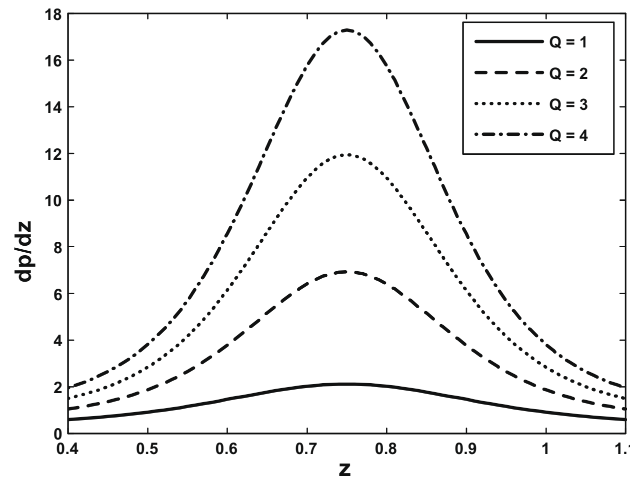

Fig. 10. Axial pressure gradient dp/dz when $\varepsilon = 0.3$, $\alpha = 0.4$, $\Gamma = 0.2$, $\phi = 0.3$.

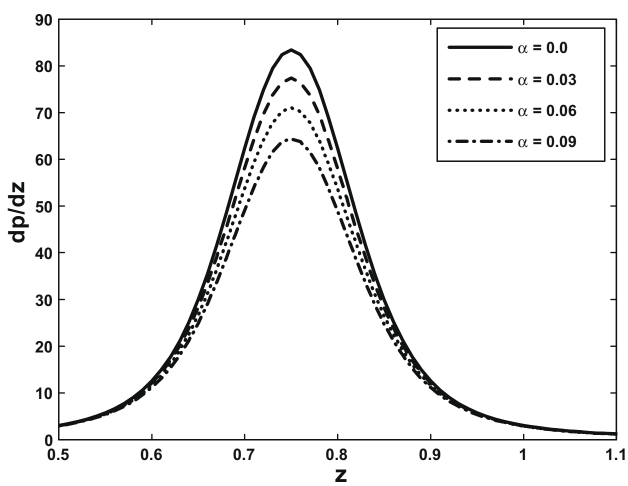

Fig. 8. Axial pressure gradient dp/dz when $\varepsilon = 0.3$, $\phi = 0.4$, $\Gamma = 0.2$, $Q = 3$.

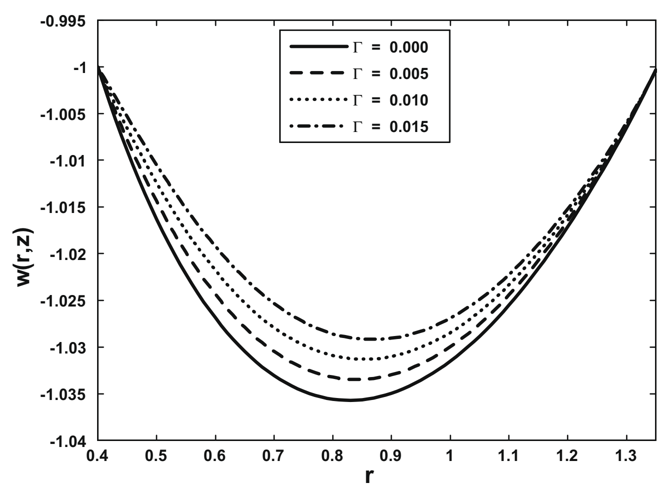

Fig. 11. Variation of Deborah number on w when $\varepsilon = 0.1, \alpha = 0.2, \frac{dp}{dz} = 0.4, z = 0.5$.

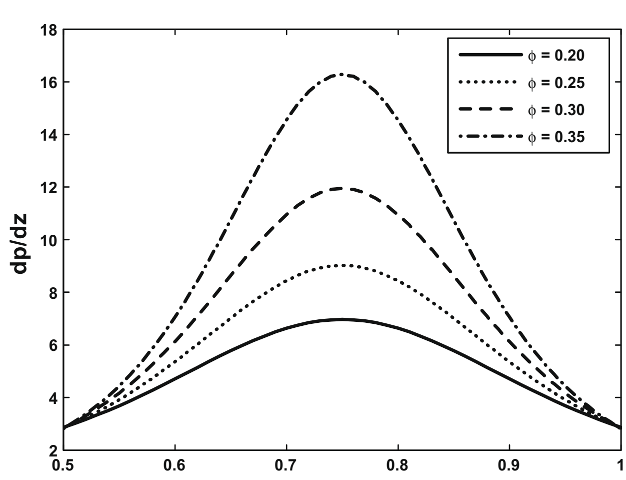

Fig. 9. Axial pressure gradient dp/dz when $\varepsilon = 0.3$, $\alpha = 0.4$, $\Gamma = 0.2$, $Q = 3$.

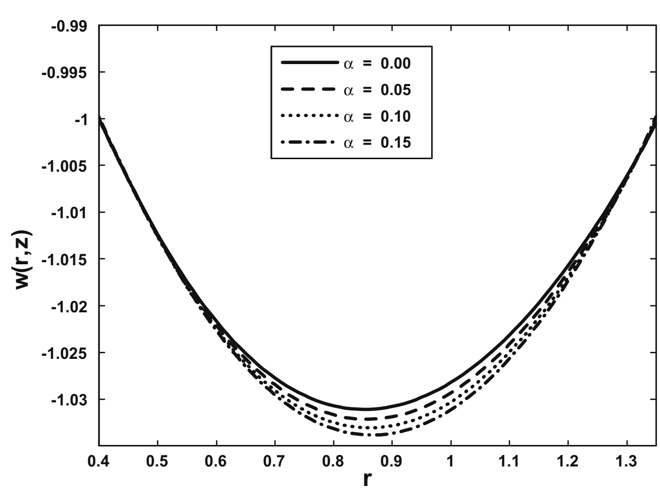

Fig. 12. Variation of viscosity parameter on w when $\varepsilon = 0.1, \Gamma = 0.01, \frac{dp}{dz} = 0.4$, $z = 0.5$.

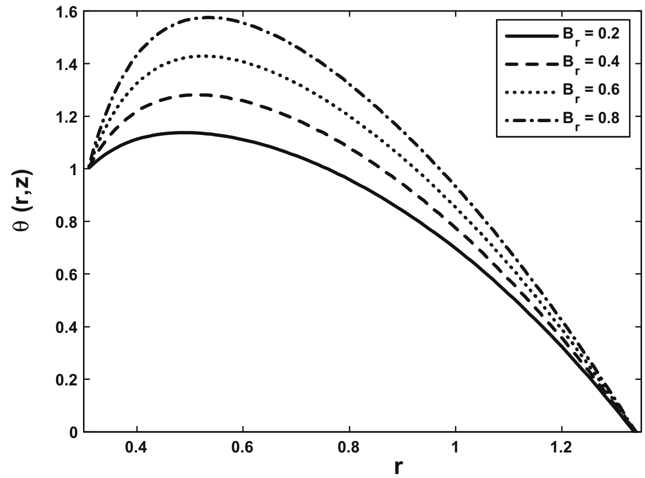

Fig. 13. Temperature profile for $\varepsilon = 0.3$, $\alpha = 0.8$, $\frac{dp}{dz} = 0.4, z = 0.1, Q = 1, \Gamma = 0.1$.

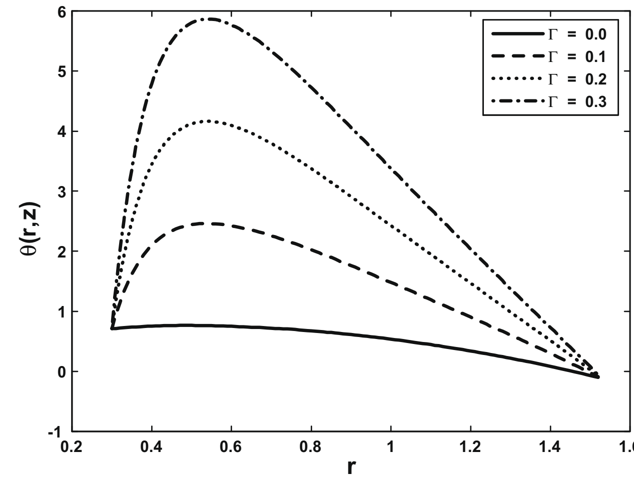

Fig. 15. Temperature profile for $\varepsilon = 0.3$, $B_r = 0.8$, $\frac{dp}{dz} = 0.4, z = 0.1, Q = 1, \alpha = 0.1$.

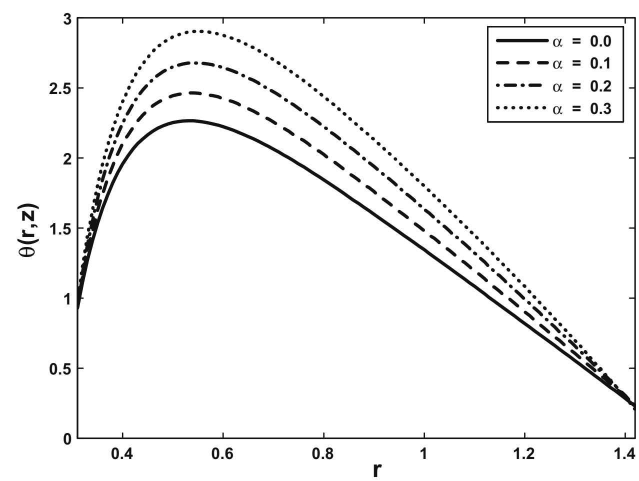

Fig. 14. Temperature profile for $\varepsilon = 0.3$, $B_r = 0.8$, $\frac{dp}{dz} = 0.4, z = 0.1, Q = 1, \Gamma = 0.1$.

solution is also compared with the perturbation solution. The difference between the values of two solutions is given in Tables 1 and 2.

5. Numerical results and discussion

In this section the pressure rise, frictional forces, axial pressure gradient, axial velocity and temperature are analyzed carefully. For this object, the Figs. 2–15 are displayed. The pressure rise is calculated numerically by using Mathematica. Figs. 2 and 3 show the pressure rise Δp against volume flow rate Q for different values of Deborah number Γ and viscosity parameter α. These Figs. indicate that the relation between pressure rise and volume flow rate are inversely proportional to each other. As expected that pressure rise gives larger values for small volume flow rate and it gives smaller values for large Q. Moreover, the peristaltic pumping occurs in the region $-1 \leqslant Q < 1$ for Fig. 2, $-3 \leqslant Q \leqslant 1$ for Fig. 3, other wise augmented pumping occurs. Moreover, the pressure rise increase with an increase in Deborah number and decreases when α is increased. Figs. 4–6 describe the variation of frictional forces. These Figs. show that the frictional forces have quite opposite features when compared with the pressure rise. The variations of pressure gradient are plotted in Figs. 7–10. Fig. 7 elucidates that pressure gradient is small when $z\varepsilon[0.5, 0.6]$ and $z\varepsilon[0.9, 1.1]$ and large when $z\varepsilon[0.61, 0.89]$. Moreover the pressure gradient increases by increasing Deborah number. Fig. 8 illustrates that the pressure gradient is small when $z\varepsilon[0.5, 0.6]$ and $z\varepsilon[0.9, 1.1]$ and large when $z\varepsilon[0.61, 0.89]$. It is also observed that pressure gradient decreases by increasing α. Figs. 9 and 10 show large pressure gradient as compared to the Figs. 7 and 8. It is also observed from the Figs. that pressure gradient increases by increasing ϕ and Q.

The velocity field w for different values of Γ and α are shown in the Figs. 11 and 12 respectively. It is noticed that axial velocity increases with an increase in Γ and decreases by increasing α (see Figs. 11 and 12). The temperature field for different values of B_r, Γ and α against space variable r are displayed in Figs. 13–15. It is found that the temperature increases when B_r, Γ and α are increased.

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Appendix A

Here we present the values of all functions appearing in the solutions (22) to (24) i.e.

$$L_1 = \frac{3(r_1^2 - r_2^2) + 2\alpha(r_1^3 - r_2^3)}{12},\ L_2 = \ln r_1 - \ln r_2,\ L_3 = -\frac{L_1}{L_2},$$

$$L_4 = \frac{-(3r_1^2 - 2\alpha r_1^3) - 12L_3(\ln r_1 + \alpha r_1)}{12},\ L_5 = 8\alpha L_3 + 4\alpha^2 L_3,$$

$$L_6 = 4L_3 + 4\alpha^2 L_3^2,\ L_7 = 8\alpha L_3^2,\ L_8 = 4L_3^2,\ L_9 = -E_c \Pr\left(\frac{dp}{dz}\frac{1}{2}\right)^2,$$

$$L_{10} = -L_9\alpha^3,\ L_{11} = -L_9\alpha^2,\ L_{12} = \alpha L_9,\ L_{13} = L_9(1 - \alpha L_5),$$

$$L_{14} = L_9(L_5 - \alpha L_6),\ L_{15} = L_9(L_6 - \alpha L_7),\ L_{16} = L_9(L_7 - \alpha L_8),$$

$$L_{17} = L_8 L_9,\ L_{18} = \frac{L_{10}}{49},\ L_{19} = \frac{L_{11}}{36},\ L_{20} = \frac{L_{12}}{25},\ L_{21} = \frac{L_{13}}{16},$$

$$L_{22} = \frac{L_{14}}{9},\ L_{23} = \frac{L_{15}}{4},\ L_{24} = \frac{L_{17}}{2},$$

$$L_{25} = L_{18}r_1^7 + L_{19}r_1^6 + L_{20}r_1^5 + L_{21}r_1^4 + L_{22}r_1^3 + L_{23}r_1^2 + L_{16}r_1 + L_{24}(\ln r_1)^2,$$

$$L_{26} = L_{18}r_2^7 + L_{19}r_2^6 + L_{20}r_2^5 + L_{21}r_2^4 + L_{22}r_2^3 + L_{23}r_2^2 + L_{16}r_2 + L_{24}(\ln r_2)^2,$$

$$L_{27} = \frac{1 - L_{25} + L_{26}}{\ln\frac{r_1}{r_2}},\ L_{28} = 1 - L_{25} - L_{27}\ln r_1,\ L_{29} = \left(\frac{\partial p}{\partial z}\frac{1}{2}\right)^3,$$

$$L_{30} = L_{29}\alpha^3,\ L_{31} = -L_{29}\alpha^2,\ L_{32} = (3\alpha + 2L_3\alpha^3)\,L_{29},$$

$$L_{33} = L_{29}(1 + \alpha L5 + 6\alpha^2 L_3),\ L_{34} = L_{29}(L_5 + \alpha L_6 + 6\alpha L_3),$$

$$L_{35} = L_{29}(L_6 + \alpha L_7 + 2L_3 + 2L_3L_5\alpha),$$
$$L_{36} = L_{29}(L_7 + \alpha L_8 + 2L_3L_5 + 2L_3L_6\alpha),$$

$$L_{37} = L_{29}(L_8 + 2L_3L_6 + 2L_3L_7\alpha),\ L_{38} = (2L_3L_7 + 2L_3L_8\alpha),$$
$$L_{39} = 2L_3L_8,$$

$$L_{40} = -2\alpha L_{30},\ L_{41} = (-2L_{30} - 2\alpha L_{31}),\ L_{42} = (-2L_{31} - 2\alpha L_{32}),$$

$$L_{43} = (-2L_{32} - 2\alpha L_{33}),\ L_{44} = (-2L_{33} - 2\alpha L_{34}),$$
$$L_{45} = (-2L_{34} - 2\alpha L_{35}),$$

$$L_{46} = (-2L_{35} - 2\alpha L_{36}),\ L_{47} = (-2L_{36} - 2\alpha L_{37}),$$
$$L_{48} = (-2L_{37} - 2\alpha L_{38}),$$

$$L_{49} = (-2L_{38} - 2\alpha L_{39}),\ L_{50} = -2L_{39},$$

$$L_{51} = L_{56}r_1^8 + L_{57}r_1^7 + L_{58}r_1^6 + L_{59}r_1^5 + L_{60}r_1^4 + L_{61}r_1^3 + L_{62}r_1^2 + L_{47}r_1 + L_{48}\ln r - \frac{L_{49}}{r_1} - \frac{L_{50}}{2r_1^2},$$

$$L_{52} = L_{56}r_2^8 + L_{57}r_2^7 + L_{58}r_2^6 + L_{59}r_2^5 + L_{60}r_2^4 + L_{61}r_2^3 + L_{62}r_2^2 + L_{47}r_2 + L_{48}\ln r_2 - \frac{L_{49}}{r_2} - \frac{L_{50}}{2r_2^2},$$

$$L_{53} = L_{51} - L_{52},\ L_{54} = -\frac{L_{53}}{L_2},\ L_{55} = -L_{54}(\ln r_1 + \alpha r_1) - L_{51},$$

$$L_{56} = \frac{L_{40}}{8},$$

$$L_{57} = \frac{L_{41}}{7},\ L_{58} = \frac{L_{42}}{6},\ L_{59} = \frac{L_{43}}{5},\ L_{60} = \frac{L_{44}}{4},\ L_{61} = \frac{L_{45}}{3},\ L_{62} = \frac{L_{46}}{2},$$

$$L_{63} = -\frac{L_{50}}{2},\ L_{64} = \alpha L_{54} + L_{47},\ L_{65} = \alpha L_{55} + L_{48},$$

$$L_{66} = \frac{1}{120}\Bigg(15(r_2^4 - r_1^4) + \frac{2\alpha}{5}(r_2^5 - r_1^5) + 20L_3\big(6(r_2^2\ln r_2 - r_1^2\ln r_1) - 3(r_2^2 - r_1^2) + +4\alpha(r_2^3 - r_1^3)\big) + 120L_4(r_2^2 - r_1^2)\Bigg),$$

$$L_{67} = \frac{1}{5}L_{56}(r_2^{10} - r_1^{10}) + \frac{2}{9}L_{57}(r_2^9 - r_1^9) + \frac{1}{4}L_{58}(r_2^8 - r_1^8) + \frac{2}{7}L_{59}(r_2^7 - r_1^7) + \frac{1}{3}L_{60}(r_2^6 - r_1^6) + \frac{2}{5}L_{61}(r_2^5 - r_1^5) + \frac{1}{2}L_{62}(r_2^4 - r_1^4) + \frac{2}{3}L_{64}(r_2^3 - r_1^3) + L_{55}(r_2^2 - r_2^2) + 2L_{65}\left(\frac{(r_2^2\ln r_2 - r_1^2\ln r_1)}{2} - \frac{(r_2^2 - r_1^2)}{4}\right) - 2L_{49}(r_2 - r_1) + 2L_{63}(\ln r_2 - \ln r_1),$$

$$L_{68} = \alpha L_{30},\quad L_{69} = L_{30} + \alpha L_{31},\quad L_{70} = L_{31} + \alpha L_{32} + 2\alpha L_3L_{30},$$

$$L_{71} = L_{32} + \alpha L_{33} + 2L_3L_{30} + 2\alpha L_3L_{31},$$
$$L_{72} = L_{33} + \alpha L_{34} + 2L_3L_{31} + 2\alpha L_3L_{32},$$

$$L_{73} = L_{34} + \alpha L_{35} + 2L_3L_{32} + 2\alpha L_3L_{33},$$
$$L_{74} = L_{35} + \alpha L_{36} + 2L_3L_{33} + 2\alpha L_3L_{34},$$

$$L_{75} = L_{36} + \alpha L_{37} + 2L_3L_{34} + 2\alpha L_3L_{35},$$
$$L_{76} = L_{37} + \alpha L_{38} + 2L_3L_{35} + 2\alpha L_3L_{36},$$

$$L_{77} = L_{38} + \alpha L_{39} + 2L_3L_{36} + 2\alpha L_3L_{37},\ L_{78} = L_{39} + 2L_3L_{37} + 2\alpha L_3L_{38},$$

$$L_{79} = 2L_3L_{38} + 2\alpha L_3L_{39},\ L_{80} = 2L_3L_{39},\ L_{81} = 64(L_{56})^2,$$
$$L_{82} = 256L_{56}L_{57},$$

$$L_{83} = 248L_{56}L_{58} + 49(L_{57})^2,\ L_{84} = 80L_{56}L_{59} + 84L_{57}L_{58},$$

$$L_{85} = 64L_{60}L_{56} + 70L_{57}L_{59} + 36(L_{58})^2,$$

$$L_{86} = 48L_{61}L_{56} + 56L_{57}L_{60} + 60L_{58}L_{59} + \left(\frac{dp_1}{dz}\frac{1}{2}\right)(24\alpha L_{56}),$$

$$L_{87} = 32L_{62}L_{56} + 42L_{57}L_{61} + 48L_{58}L_{60} + 25(L_{59})^2 + \left(\frac{dp_1}{dz}\frac{1}{2}\right) \times (16L_{56} + 21\alpha L_{57} + 16\alpha L_{56}L_3),$$

$$L_{88} = 16L_{64}L_{56} + 28L_{57}L_{62} + 36L_{58}L_{61} + 40L_{59}L_{60} + \left(\frac{dp_1}{dz}\frac{1}{2}\right) \times (14L_{57} + 18\alpha L_{58} + 16\alpha L_{56}L_3),$$

$$L_{89} = 8L_{65}L_{56} + 14L_{57}L_{64} + 24L_{58}L_{62} + 30L_{59}L_{61} + 16(L_{60})^2 + \left(\frac{dp_1}{dz}\frac{1}{2}\right)(12L_{58} + 15\alpha L_{59} + 32L_{56}L_3 + 28\alpha L_{57}L_3),$$

$$L_{90} = 8L_{49}L_{56} + 7L_{57}L_{65} + 16L_{58}L_{64} + 20L_{59}L_{62} + 24L_{60}L_{61} + \left(\frac{dp_1}{dz}\frac{1}{2}\right)(10L_{59} + 12\alpha L_{60} + 28L_{57}L_3 + 24\alpha L_{58}L_3),$$

$$L_{91} = -16L_{63}L_{56} + 7L_{57}L_{49} + 6L_{58}L_{65} + 10L_{59}L_{64} + 16L_{60}L_{62} + 9(L_{61})^2 + \left(\frac{dp_1}{dz}\frac{1}{2}\right)(8L_{60} + 9\alpha L_{61} + 24L_{58}L_3 + 20\alpha L_{59}L_3),$$

$$L_{92} = -14L_{63}L_{57} + 6L_{58}L_{49} + 5L_{59}L_{65} + 8L_{60}L_{64} + 12L_{61}L_{62} + 2\alpha\left(\frac{dp_1}{dz}\frac{1}{2}\right)^2 + \left(\frac{dp_1}{dz}\frac{1}{2}\right)(5L_{61} + 6\alpha L_{62} + 20L_{59}L_3 + 16\alpha L_{60}L_3),$$

$$L_{93} = -12L_{63}L_{58} + 5L_{59}L_{49} + 4L_{60}L_{65} + 3L_{61}L_{64} + 3L_{61}L_{64} + 4(L_{62})^2 + (1 + 4L_3\alpha^2)\left(\frac{dp_1}{dz}\frac{1}{2}\right)^2 + \left(\frac{dp_1}{dz}\frac{1}{2}\right) \times (4L_{62} + 4\alpha L_{64} + 16L_{60}L_3 + 12\alpha L_{61}L_3),$$

$$L_{94} = -10L_{63}L_{59} + 4L_{60}L_{49} + 3L_{61}L_{65} + 2L_{62}L_{63} + 2L_{62}L_{64} + (8L_3\alpha)\left(\frac{dp_1}{dz}\frac{1}{2}\right)^2 + \left(\frac{dp_1}{dz}\frac{1}{2}\right) \times (2L_{64} + 3\alpha L_{65} + 12L_{61}L_3 + 8\alpha L_{62}L_3),$$

$$L_{95} = -8L_{63}L_{60} + 3L_{62}L_{49} + 3L_{62}L_{65} + (L_{64})^2 + (4L_3 + 4L_3^2\alpha^2)\left(\frac{dp_1}{dz}\frac{1}{2}\right)^2 + \left(\frac{dp_1}{dz}\frac{1}{2}\right)(2L_{65} + 3\alpha L_{49} + 8L_{62}L_3 + 4\alpha L_{64}L_3 - 2L_{63}\alpha),$$

$$L_{96} = 2L_{62}L_{49} + L_{65}L_{64} - 6L_{63}L_{61} + (2\alpha L_3 + 4L_3^2\alpha)\left(\frac{dp_1}{dz}\frac{1}{2}\right)^2 + \left(\frac{dp_1}{dz}\frac{1}{2}\right)(2L_{49} + 4L_{64}L_3 + 4\alpha L_{65}L_3 - 4L_{63}\alpha),$$

$$L_{97} = -2L_{62}L_{63} + L_{49}L_{64} + \left(\frac{dp_1}{dz}\frac{1}{2}\right) \times (-4L_{63} + 4L_{65}L_3 + 4\alpha L_{49}L_3 - 4L_{63}\alpha),$$

$$L_{98} = -2L_{64}L_{63} + \left(\frac{dp_1}{dz}\frac{1}{2}\right)(-6L_{63}L_3\alpha + 4L_{49}L_3),$$

$$L_{99} = \left(\frac{dp_1}{dz}\frac{1}{2}\right)(-8L_{63}L_3\alpha),$$

$$L_{100} = E_c\Pr\alpha L_{81},\ L_{101} = -E_c\Pr(L_{81} - \alpha L_{82}),\ L_{102} = -E_c\Pr(L_{82} - \alpha L_{83}),$$

$$L_{103} = -E_c\Pr(L_{83} - \alpha L_{84}),\ L_{104} = -E_c\Pr(L_{84} - \alpha L_{85}),$$
$$L_{105} = -E_c\Pr(L_{85} - \alpha L_{86}),$$

$$L_{106} = -E_c\Pr(L_{86} - \alpha L_{87}),\ L_{107} = -E_c\Pr(L_{87} - \alpha L_{88} + 2L_{68}),$$

$$L_{108} = -E_c\Pr(L_{88} - \alpha L_{89} + 2L_{69}),\ L_{109} = -E_c\Pr(L_{89} - \alpha L_{90} + 2L_{70}),$$

$$L_{110} = -E_c\Pr(L_{90} - \alpha L_{91} + 2L_{71}),\ L_{111} = -E_c\Pr(L_{91} - \alpha L_{92} + 2L_{72}),$$

$$L_{112} = -E_c\Pr(L_{92} - \alpha L_{93} + 2L_{73}),\ L_{113} = -E_c\Pr(L_{93} - \alpha L_{94} + 2L_{74}),$$

$$L_{114} = -E_c\Pr(L_{94} - \alpha L_{95} + 2L_{75}),\ L_{115} = -E_c\Pr(L_{95} - \alpha L_{96} + 2L_{76}),$$

$$L_{116} = -E_c\Pr(L_{96} - \alpha L_{97} + 2L_{77}),\ L_{117} = -E_c\Pr(L_{97} - \alpha L_{98} + 2L_{78}),$$

$$L_{118} = -E_c\Pr(L_{98} - \alpha L_{99} + 2L_{79}),\ L_{119} = -E_c\Pr(L_{99} + 2L_{80}),$$

$$L_{120} = \frac{L_{100}}{289},\ L_{121} = \frac{L_{101}}{256},\ L_{122} = \frac{L_{102}}{225},\ L_{123} = \frac{L_{103}}{196},\ L_{124} = \frac{L_{104}}{169},$$

$$L_{125} = \frac{L_{105}}{144},\ L_{126} = \frac{L_{106}}{121},\ L_{127} = \frac{L_{107}}{100},\ L_{128} = \frac{L_{108}}{81},\ L_{129} = \frac{L_{109}}{64},$$

$$L_{130} = \frac{L_{110}}{49},\ L_{131} = \frac{L_{111}}{36},\ L_{132} = \frac{L_{112}}{25},\ L_{133} = \frac{L_{113}}{16},\ L_{134} = \frac{L_{114}}{9},$$

$$L_{135} = \frac{L_{115}}{4},\ L_{136} = \frac{L_{117}}{2},\ L_{137} = 4L_{119},$$

$$L_{138} = L_{120}r_1^{17} + L_{121}r_1^{16} + L_{122}r_1^{15} + L_{123}r_1^{14} + L_{124}r_1^{13} + L_{125}r_1^{12} + L_{126}r_1^{11} + L_{127}r_1^{10} + L_{128}r_1^{9} + L_{129}r_1^{8} + L_{130}r_1^{7} + L_{131}r_1^{6} + L_{132}r_1^{5} + L_{133}r_1^{4} + L_{134}r_1^{3} + L_{135}r_1^{2} + L_{116}r_1 + L_{136}(\ln r_1)^2 + \frac{L_{118}}{r_1} + \frac{L_{137}}{r_1^2},\ L_{140} = \frac{L_{138} - L_{139}}{\ln r_2 - \ln r_1}$$

$$L_{139} = L_{120}r_2^{17} + L_{121}r_2^{16} + L_{122}r_2^{15} + L_{123}r_2^{14} + L_{124}r_2^{13} + L_{125}r_2^{12} + L_{126}r_2^{11} + L_{127}r_2^{10} + L_{128}r_2^{9} + L_{129}r_2^{8} + L_{130}r_2^{7} + L_{131}r_2^{6} + L_{132}r_2^{5} + L_{133}r_2^{4} + L_{134}r_2^{3} + L_{135}r_2^{2} + L_{116}r_2 + L_{136}(\ln r_2)^2 + \frac{L_{118}}{r_2} + \frac{L_{137}}{r_2^2},\ L_{141} = -L_{138} - L_{141}\ln r_1,$$

$$a_1(r) = L_{18}r^7 + L_{19}r^6 + L_{20}r^5 + L_{21}r^4 + L_{22}r^3 + L_{23}r^2 + L_{16}r + L_{24}(\ln r)^2 + L_{27}\ln r + L_{28} + \Gamma\big(L_{120}r^{17} + L_{121}r^{16} + L_{122}r^{15} + L_{123}r^{14} + L_{124}r^{13} + L_{125}r^{12} + L_{126}r^{11} + L_{127}r^{10} + L_{128}r^{9} + L_{129}r^{8} + L_{130}r^{7} + L_{131}r^{6} + L_{132}r^{5} + L_{133}r^{4} + L_{134}r^{3} + L_{135}r^{2} + L_{116}r + L_{136}(\ln r)^2 + \frac{L_{118}}{r} + \frac{L_{137}}{r^2} + L_{140}\ln r + L_{141},$$

$$a_2(r) = \frac{r}{2}a_1(r),\ a_3(r) = \frac{a_1(r)}{r},\ a_4(r) = \int a_2(r)dr,\ a_5(r) = \int a_3(r)dr,$$

$$a_6(r) = w_0(r),\ a_7(r) = -2\int a_2(r)dr,\ a_{11} = \frac{r_1^2}{4} - \frac{r_2^2}{4} + \beta(a_4(r_1) - a_4(r_2)),$$

$$a_8(r) = \int_{r_1}^{r_2}\left(\frac{r^2}{4} + \beta a_4(r) - \frac{a_{11}}{a_{12}}(\ln r + \beta a_5(r) - a_{13})\right)dr,\ a_{10}(r) = (a_9(r))^2,$$

$$a_9(r) = \left(\frac{dp}{dz}\right)^3\left(\frac{r}{2} + \beta a_2(r) - \frac{a_{11}}{a_{12}}\left(\frac{1}{r} + \beta a_3(r)\right)\right),\ a_{11}(r) = \beta r a_{10}(r)a_1(r),$$

$$a_{12}(r) = \int r a_{10}(r)dr,\ a_{13}(r) = \int a_{11}(r)dr,$$
$$a_{14}(r) = \int\frac{a_{12}(r)}{r}dr,\ a_{15}(r) = \int\frac{a_{13}(r)}{r}dr,$$

$$a_{16}(r) = \left(\frac{\partial w_1}{\partial r}\right),\ a_{17}(r) = (a_{16}(r))^2,\ a_{18}(r) = (a_9(r))^4,$$
$$a_{20}(r) = -\beta r a_1(r)a_{17}(r),$$

$$a_{21}(r) = a_{17}(r) + a_{18}(r),\ a_{22}(r) = \int r a_{21}(r)dr,$$
$$a_{23}(r) = \int B_r a_{20}(r)dr,$$

$$a_{24}(r) = \int\frac{a_{22}(r)}{r}dr,\ a_{25}(r) = \int\frac{a_{23}(r)}{r}dr,$$

$$a_{12} = \ln r_1 - \ln r_2 + \beta(a_5(r_1) - a_5(r_2)),\ a_{15} = -\frac{a_{14}}{a_{11}} - a_7(r_1).$$

$$a_{13} = \left(\frac{r_1^2}{4} + \beta a_4(r_1) - \frac{a_{11}}{a_{12}}(\ln r_1 + \beta a_5(r_1) - a_{13})\right),$$
$$a_{14} = a_7(r_1) - a_7(r_2),$$

$$a_{16} = \frac{1 + B_r(a_{14}(r_1) + a_{15}(r_1) - a_{14}(r_2) - a_{15}(r_2))}{\ln r_1 - \ln r_2},$$
$$a_{17} = 1 + B_r(a_{14}(r_1) + a_{15}(r_1)) - a_{16}\ln r_1,$$

$$a_{18} = \frac{-B_r}{\ln r_1 - \ln r_2}(a_{24}(r_2) - a_{25}(r_1) - a_{24}(r_2) + a_{15}(r_2)),$$

$$a_{19} = B_r(a_{24}(r_1) + a_{25}(r_1) - a_{18}\ln r_1).$$

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